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Fluctuating lift and drag on a long cylinder of square cross-section in a smooth and in a turbulent stream

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The paper presents the results of measurements of fluctuating lift and drag on a long square cylinder. The measurements include the correlation of lift along the cylinder and the distribution of fluctuating pressure on a cross-section. The magnitude of the fluctuating lift was found to be considerably greater than that for a circular cross-section and the spanwise correlation much stronger.

It was found that the presence of large-scale turbulence in the stream had a marked influence on both the steady and the fluctuating forces. The most significant changes were at small angles of attack ($\alpha < 10^\circ$) and included a reduction in base suction and a decrease in fluctuating lift of about 50 %.

1. Introduction

The work described in this paper forms part of a study of the fluctuating loads on bluff structures in turbulent flow. The study is directed towards the assessment of wind loads and wind effects on buildings and the scale and intensity of turbulence were chosen to be roughly consistent with this application. While the fluctuating lift forces and the associated vortex shedding from a circular cylinder have been the subject of considerable research, comparatively little attention has been paid to other bluff shapes. Little is known of the effects of large-scale turbulence on vortex shedding from bluff structures although this is a problem of considerable practical significance.

All measurements were made on a 6 in. × 6 in. square cylinder spanning the working section of a 7 ft. × 7 ft. low-speed wind tunnel. The turbulence was produced by a grid of bars, $4\frac{1}{2}$ in. wide at 21 in. centres. The grid and the structure of the turbulence produced by it are described in detail elsewhere (Vickery 1965). At the test position of the cylinder the longitudinal scale of turbulence was approximately 8 in. and the intensity 10 %. The scale of atmospheric turbulence in strong winds at a height of, say, 300 ft. above an urban area would be of the order of a few hundred feet and hence the model employed in the tests is, in size, roughly representative of a large building. The intensity of turbulence is representative of atmospheric conditions at heights clear of the general roughness projections of the terrain.

The notation used throughout the paper is as follows:

D length of a side of a square cylinder;
$\overline{U}$ mean velocity;
n frequency;

C_p pressure coefficient;
C_{pb} base-pressure coefficient;
C_{pf} R.M.S. coefficient of pressure fluctuations;
C_{L_f} R.M.S. coefficient of lift fluctuations;
C_{D_f} R.M.S. coefficient of drag fluctuations;
ρ density of air;
p pressure;
$\Phi(n)$ spectral density;
R correlation coefficient;
S Strouhal number;
α angle of attack;
r a linear displacement;
$R_e(\overline{U}D/\nu)$ Reynolds number;
ν kinematic viscosity.

2. Experimental procedure

The cylinder, of 6 in. × 6 in. square cross-section, was mounted transversely across the working section of a 7 ft. × 7 ft. low-speed wind tunnel. The cylinder was supported on vertical steel frames attached to the floor of the tunnel through rubber isolating pads. Measurements were made of the fluctuating load on a 3 in. section of the cylinder using a uni-directional strain-gauge dynamometer ('Ether' type UF/1, range ± 2 lb.) mounted within the cylinder (figure 1). The gap between the 'live' section and the dummy ends was set at 0·010 in. The natural frequency of the rig was approximately 180 cycles/sec and this permitted measurements of response up to about 50 cycles/sec or values of $nD/\overline{U}$ up to 1·0.

In addition to the load on a segment, measurements were made of the surface-pressure fluctuations. The cylinder was fitted with two sets of pressure tappings; each set comprised pairs in which one tapping was directly opposite the other. This arrangement allowed the direct measurement of the fluctuating pressure difference across the cylinder. One set of tappings was around a cross-section and the second set was at irregular intervals along the centre line of the cylinder. The latter set was used to measure the spanwise correlation of lift fluctuations. Pressures were measured on a capacitance-type pressure gauge which, with the associated pressure tubing, had a flat frequency response to about 20 cycles/sec or $nD/\overline{U} \simeq 0{\cdot}3$.

The strain-gauge balance was a body-axis system and hence at other than $\alpha = 0°$ it was not possible to measure actual lift. The term 'lift' will be used to describe the force normal to the faces nearest to being parallel to the direction of flow. Measurements of the lift fluctuations were made for $\alpha = 0$, $7\frac{1}{2}$, 20, 30 and 45° in both a smooth and a turbulent stream. The output from the transducer was recorded on magnetic tape for spectral analysis. The fluctuating drag force was measured and analysed at $\alpha = 0°$ only. In addition to the force measurements, the mean base pressure was determined for $\alpha = 0$, $7\frac{1}{2}$, 20, 30 and 45°. The measurements of pressure and force coefficients and of Strouhal number were made at about six different wind speeds in a range of Reynolds number,

4×10^4 to $1{\cdot}6 \times 10^5$. Within the limits of experimental accuracy there was no variation of the coefficients or the Strouhal number with R_e and the results quoted are mean values for the range of R_e covered by the tests. The estimated standard error of an individual determination of a force or pressure coefficient at a wind speed corresponding to $R_e \simeq 10^5$ was 3% and, for the Strouhal number, 2%.

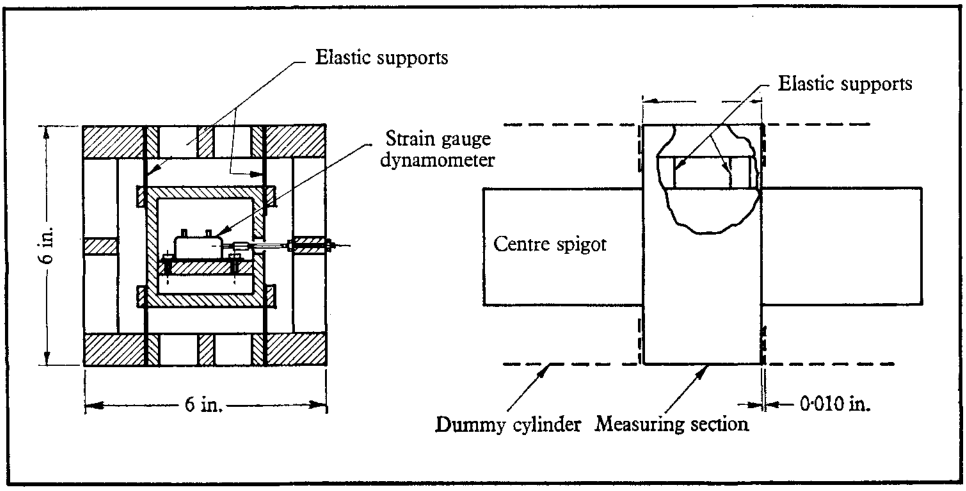

FIGURE 1. Arrangement of load-sensing element of 6 in. × 6 in. cylinder.

More detailed measurements were made at $\alpha = 0°$; these included the distribution of fluctuating pressure differences on a cross-section. The spanwise correlation was determined from the pressure difference across the centre-line of the cylinder. The correlation measurements and the spectral analysis were made at a single wind speed corresponding to a Reynolds number of $1{\cdot}0 \times 10^5$.

Spectral analysis was carried out with a 'Fenlow' low-frequency analyser. The sample for analysis was equal in length to more than 1000 cycles of shedding and this gave acceptable accuracy even for small band widths near the Strouhal frequency. The time scale was altered by use of a tape recorder to bring the sample length within the time constant of the thermocouple power meter. The peaked spectra of fluctuating lift were such as to prohibit accurate measurements of spectral density of values of $nD/\overline{U}$ greater than about 0·5. This was because the power at high frequencies was very small compared with that in the peak and pre-filter amplifiers were overloaded before the filtered signal reached a measurable level. There were no such limitations on the measurement of spectra for drag fluctuations.

3. Experimental results

The root-mean-square values of the coefficient of the lift fluctuations are presented in figure 2. These values have been adjusted for tunnel blockage and, using the spanwise correlation curve, they have been corrected to give the coefficient for load per unit length for a strip length approaching zero. The corrections for tunnel blockage were quite substantial and varied from about 10 % at $\alpha = 0°$ to a maximum at $\alpha = 45°$ of 23 %. The basis on which the blockage corrections were determined is presented in Appendix 1. The correction for the average drag and the average pressure coefficient has been well substantiated by experimental observations (Maskell 1963). The corrections for the lift fluctuations and the Strouhal number have not been verified experimentally and are used on

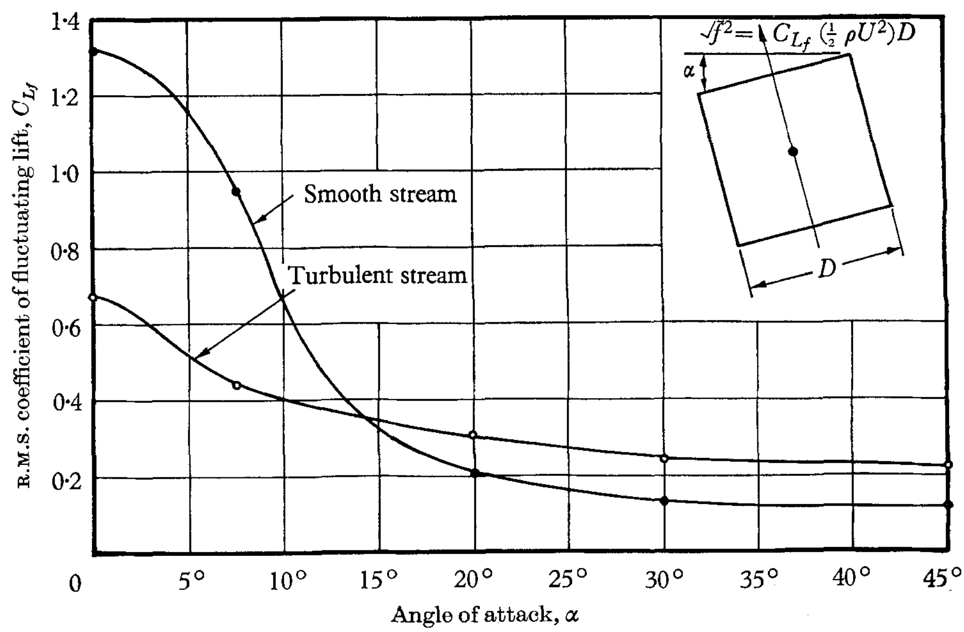

FIGURE 2. Variation of the coefficient of fluctuating lift with angle of attack.

the basis that there appear to be no more reliable methods available. The load spectra for fluctuating lift are given for $\alpha = 0°$ in figure 3; in both smooth and turbulent flow the bulk of the energy is in a very narrow band centred on the Strouhal frequency. The spectra for other angles of attack are similar in shape in that the bulk of the energy is in a narrow band. The percentage of the total energy contained in a 2 % band width centred on the Strouhal frequency is given in table 1. The Strouhal number, $nD/\overline{U}$, varied only slightly with angle of attack and the presence of turbulence had little effect. The variation of S with α is given in figure 4.

The spanwise correlation of the fluctuating pressure difference across the centre line of the model is given in figure 5. The correlation length $[\int R\,dr]$ was found to be $5{\cdot}6D$ in the case of a smooth stream and $3{\cdot}3D$ for a turbulent stream. The correlations were read directly from a calibrated ratio-meter and it was observed that in smooth flow there was a very slow change in R for values less than about 0·4. Typically, the correlation might drift from 0 to 0·6 and back over a period of a hundred or more cycles of vortex shedding. The drift was apparently due to slow changes in phase occurring continually between points greater than about

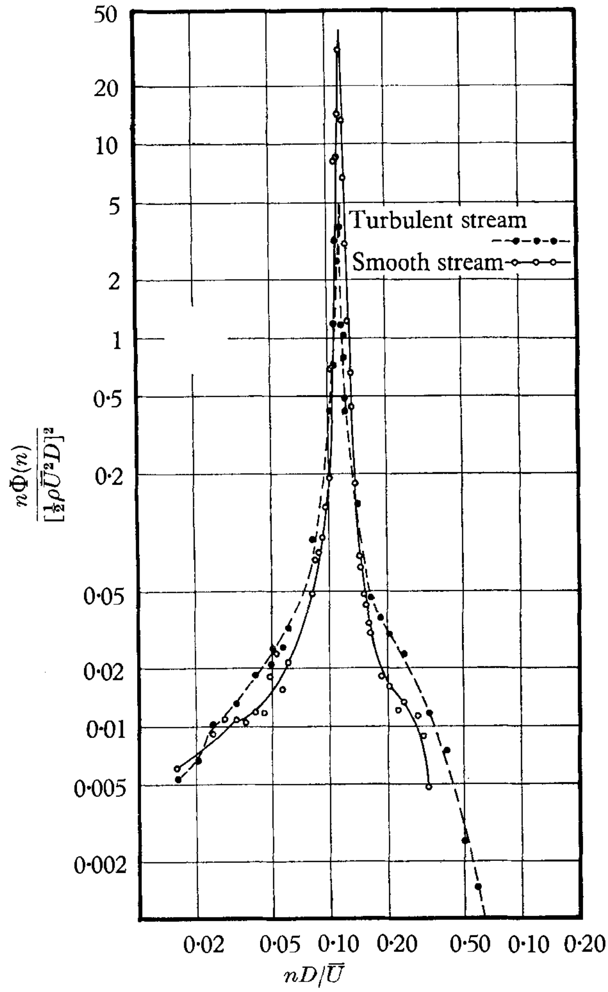

FIGURE 3. Spectrum of lift fluctuations on a square-section cylinder for $\alpha = 0°$ ($R_e = 1{\cdot}0 \times 10^5$).

Angle of attack	Percentage energy in 2 % band width centred on the Strouhal frequency	
	Smooth stream	Turbulent stream
0°	95	82
$7\frac{1}{2}$°	96	67
20°	84	73
30°	80	68
45°	78	64

TABLE 1

$7D$ apart. The presence of large-scale turbulence reduced the correlation, but the slow drift was not present to the same degree.

The distribution of the fluctuating pressure differences over a section are presented in figure 6 and the corresponding correlation measurements in figure 7. The maximum R.M.S. pressure coefficient of 1·76 corresponds to a peak to peak

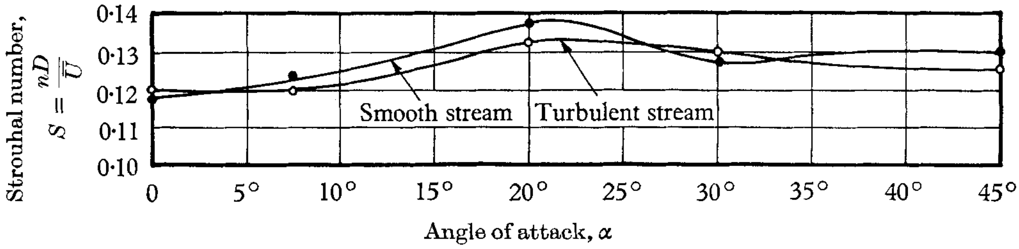

FIGURE 4. Variation of Strouhal number with angle of attack.

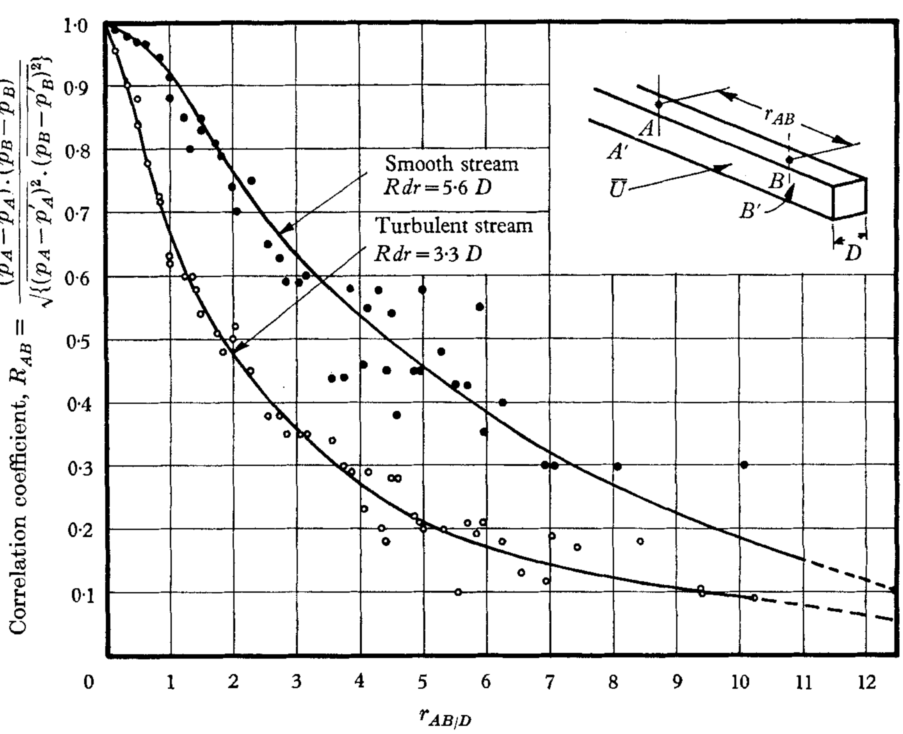

FIGURE 5. Spanwise correlation of the fluctuating pressure difference across the centre line of a long square-section cylinder for $\alpha = 0°$ ($R_e = 1{\cdot}0 \times 10^5$).

coefficient of about 6; the variation in pressure was seen to be almost sinusoidal with an amplitude which changed slowly between fairly close limits. The instantaneous pressure coefficient at a point near the centre of a face varied from about $+0{\cdot}2$ to -3. The R.M.S. coefficients of lift fluctuations computed from the pressure distribution of figure 6 and the correlations in figure 7 were 1·27 in a smooth stream and 0·63 in a turbulent stream. These might be compared with the values obtained by direct measurement, namely 1·32 and 0·68 respectively. No definite explanation can be offered for the discrepancy of about 10 % between the two methods, but the estimate based on pressures was derived from a minimum of points and is probably only accurate to within 10 %.

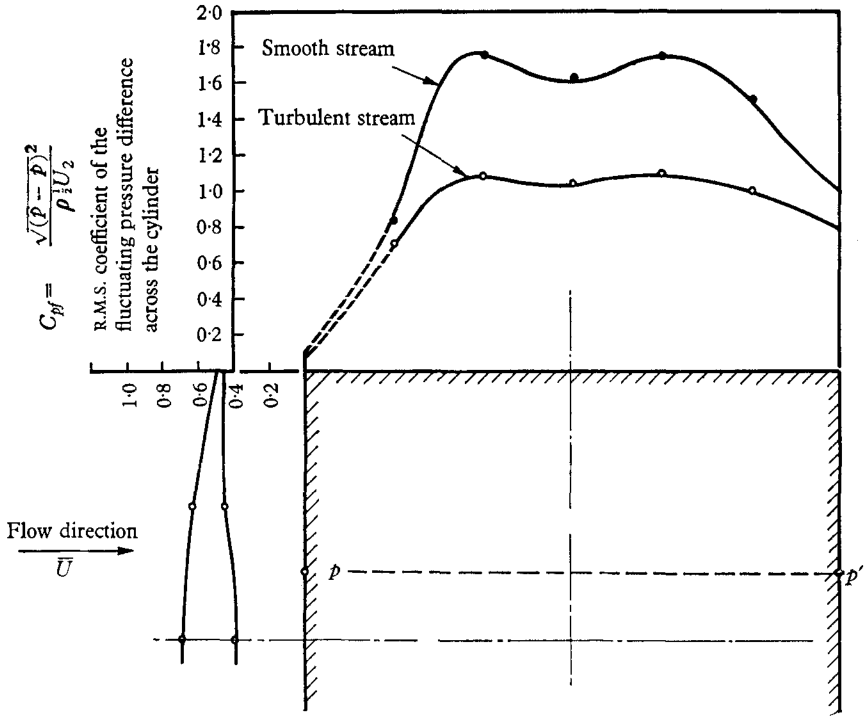

FIGURE 6. Distribution of the fluctuating pressure difference across a long cylinder of square cross-section ($\alpha = 0°$).

The base-pressure coefficients for smooth and turbulent flow are shown in figure 8. There was a substantial increase with turbulence at low angles of attack but no significant change for $\alpha > 25°$. The change in base pressure, at $\alpha = 0°$, of about 46 % would correspond to a reduction in mean drag in turbulent flow of about 25–30 %. Base pressures were measured from tappings at the mid and quarter points of the downstream face nearest to perpendicular to the direction of flow. The maximum spread of the pressures at these tappings, for a given velocity and angle of attack, was 5 % and was typically less than 2 %. The coefficients given in figure 8 are based on the mean value of the pressures at the mid and quarter points.

Measurements of fluctuating drag were made only at $\alpha = 0°$ and the load spectra for smooth and turbulent flow are given in figure 9. The root-mean-square coefficient of fluctuating drag was 0·17 in both smooth and turbulent flow, but the distribution of energy was substantially altered by the presence of turbulence. The spectral peak at $nD/\bar{U} = 2S$ was practically eliminated, while the energy at lower frequencies was increased by the loads due directly to the longitudinal component of turbulence. Both spectra had a minor peak at the normal

Strouhal frequency and another at $nD/\bar{U} = 0{\cdot}4$. The latter peak cannot be explained in terms of vortex shedding and may be due to a minor mechanical vibration in part of the model, although no definite cause could be ascertained.

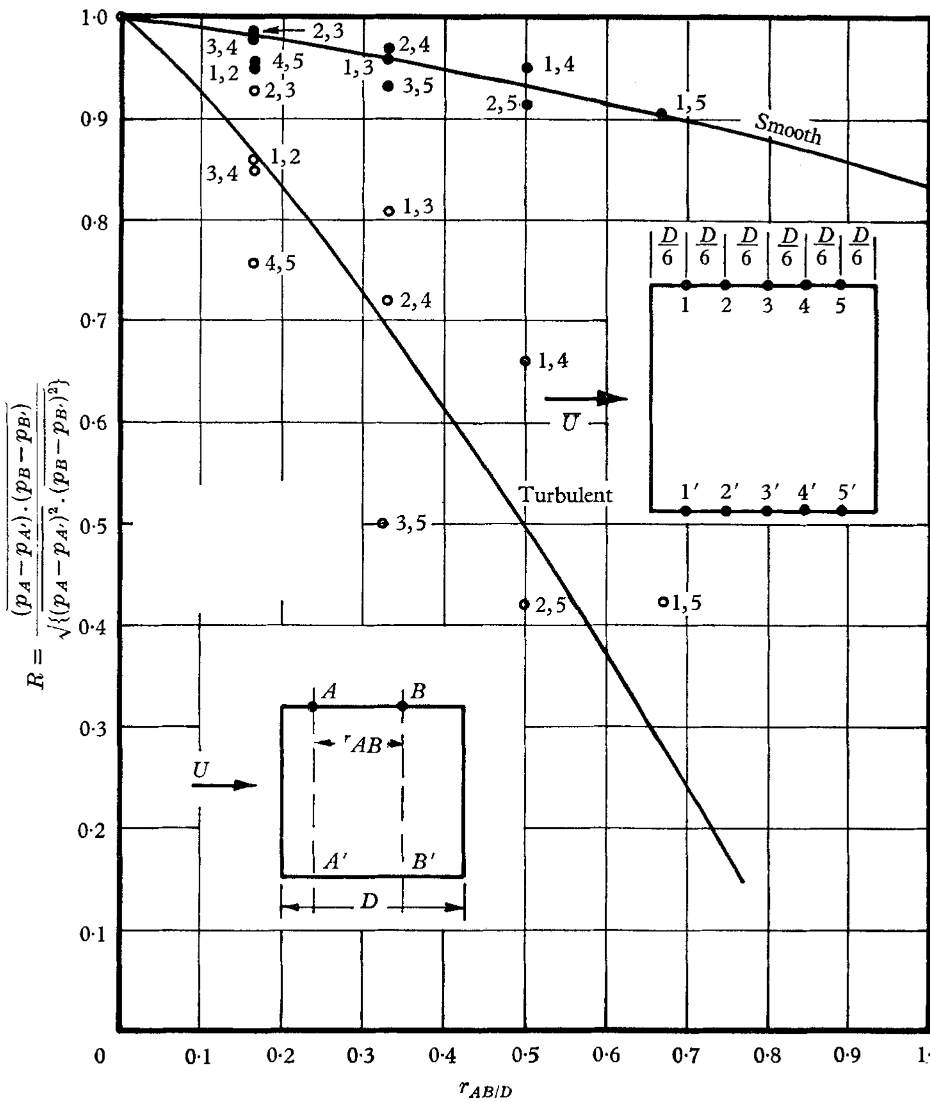

FIGURE 7. Chordwise correlation of the fluctuating pressure difference across a long cylinder of square cross-section for $\alpha = 0°$ ($R_e = 1{\cdot}0 \times 10^5$).

4. Discussion of results

Comparison with a circular cylinder

In the range of Reynolds numbers covered by the present tests the square section produces considerably greater and more strongly correlated lift forces than the circular cross-section. For Reynolds numbers between 10^4 and 10^5 various workers (MacGregor 1957; Humphreys 1960; Keefe 1962; Vickery &

Watkins 1963) have determined R.M.S. coefficients in the range 0·4 to 0·7, with most values near the lower end of this range. The fluctuating lift forces produced by the square section are then three to four times greater than those on the circular cylinder. Measurements of two-point correlations by Prendergast (1958) and el Baroudi (1960) indicate a correlation length for the circular cylinder of about $3{\cdot}5D$ as compared with the value of $5{\cdot}6D$ determined, for the square cylinder, from figure 3.

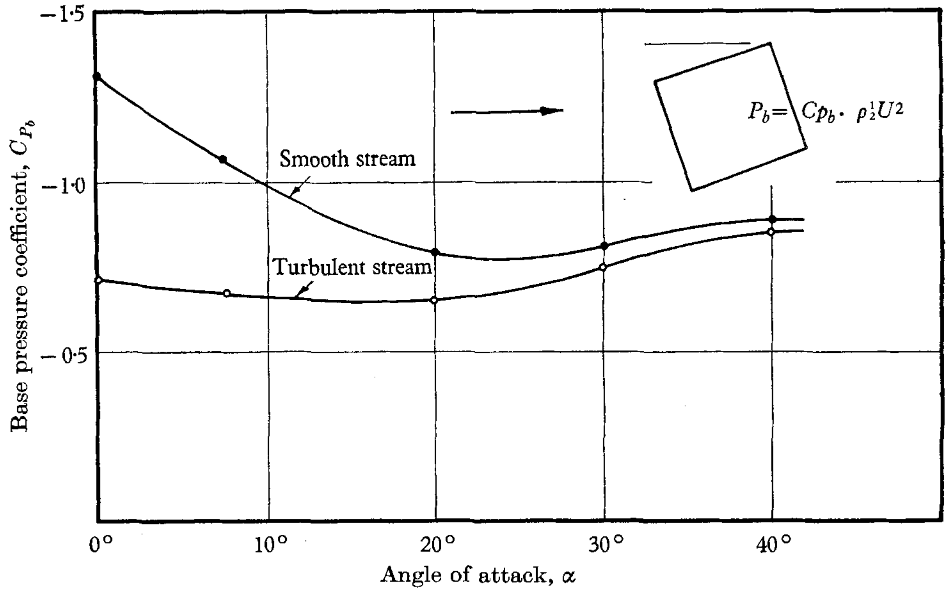

FIGURE 8. Variation of base-pressure coefficient with angle of attack in a smooth and in a turbulent stream.

The influence of turbulence

The effect of turbulence is most marked at low angles of attack. The change in base pressure is tentatively attributed to re-attachment or possibly intermittent re-attachment of the separated flow. This suggestion is supported by the fact that for large angles of attack, where there would be no possibility of re-attachment, the changes in base pressure are hardly significant. The reduction in the oscillating lift and the change in base pressure are closely related, as can be shown by considerations of the hydrodynamics of vortex wakes (Ross 1964). These considerations indicate that changes in steady drag and change in oscillating lift will be of the same order, a result which is supported by the experimental observations.

The effect of aspect ratio

The present study included no systematic investigation of the influence of aspect ratio on the magnitude of lift fluctuations, but from other sources this is known to be of major significance. Available data are primarily from tests on vibrating structures, as opposed to the stationary model employed in the present study. However, experimental results (Vickery & Watkins 1963; Vickery &

Walshe 1965) indicate that at very small amplitudes the lift fluctuations are independent of the motion. These results also show that at an aspect ratio of 30 the lift forces are only slightly different from those occurring under two-dimensional conditions, but that the strength of the fluctuations falls off rapidly

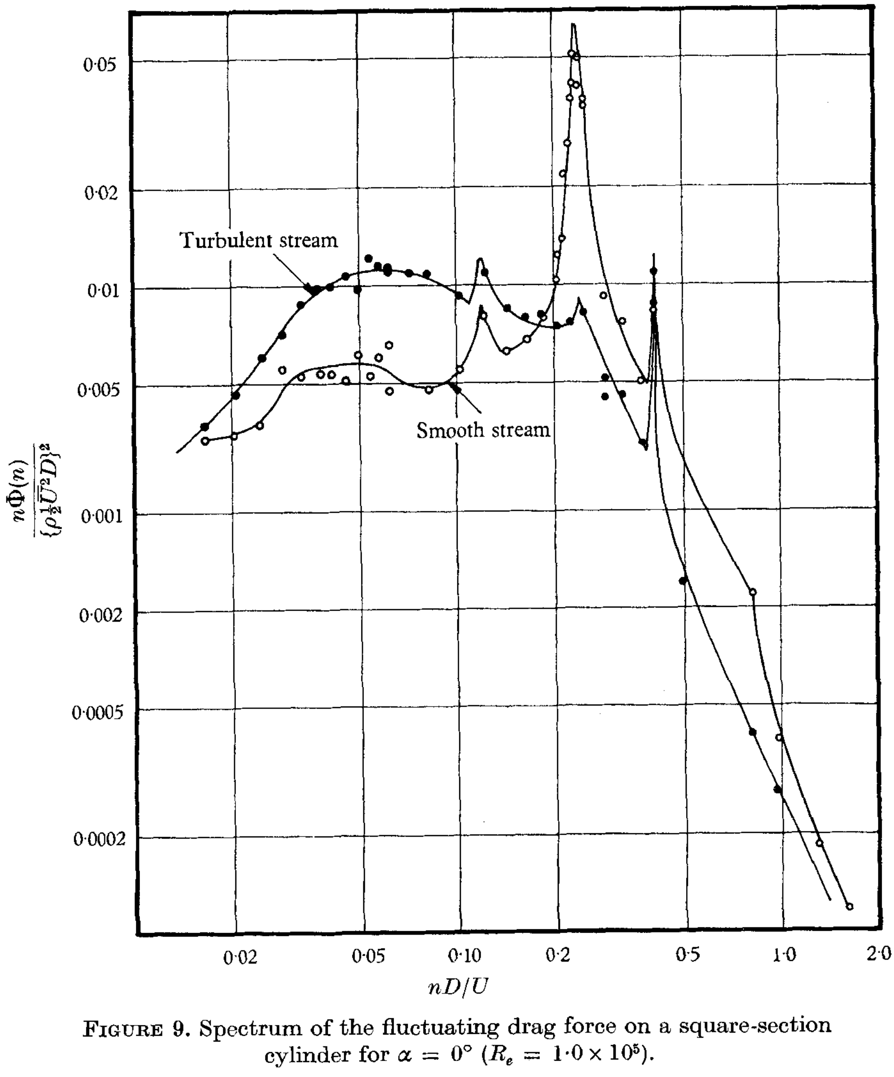

FIGURE 9. Spectrum of the fluctuating drag force on a square-section cylinder for $\alpha = 0°$ ($R_e = 1{\cdot}0 \times 10^5$).

with a decrease in aspect ratio below about 30, and at an aspect ratio of 15 the lift forces in a smooth stream are only about 20 % of that for two-dimensional flow.

Measurements of pressure fluctuations on a stationary cylinder of square cross-section and an aspect ratio of 14 have been made by Whitbread & Scruton (1965). The measurements were made on a model of 6 in. side and in the same turbulent stream used in the present study. The tests were conducted on a 'half-model' with the wall of the wind tunnel as the plane of symmetry. The R.M.S. coefficient of the fluctuating pressure-difference (across the centre line of the faces parallel with the direction of flow) was recorded at six sections along the model. The R.M.S. coefficient varied from nearly 1·0 at a point close to the base to a value of about 0·75 at the topmost tapping, which was at a distance of $\frac{1}{3}D$ from the tip. These values are not greatly different from the value of 1·04 determined under two-dimensional conditions. The spanwise correlation was, however, more markedly changed. At a spacing of $1D$ the correlation was about 0·35, at $2D$ it was about 0·15 and was practically negligible at spacings greater than $4D$. While the correlation length ($\int R dr$) is a function of position for a cylinder of finite length, the results indicate that the correlation between two points a fixed distance apart varies only slightly with actual position, and it is possible to determine an approximate average correlation length. The averaged correlation length determined from Whitbread & Scruton's measurements is $1{\cdot}0D$ as compared with a value of $3{\cdot}3D$ in two-dimensional flow.

While there is need for further measurements on cylinders of finite aspect ratio, the results available suggest that the presence of large-scale turbulence minimizes the tendency for the magnitude of lift fluctuations to decrease with a decrease in aspect ratio. The magnitude of the surface-pressure fluctuations measured under both two- and three-dimensional conditions of flow are sufficiently great to be of importance in the designs of cladding and glass on tall buildings. Based on the measured R.M.S. coefficients, it is estimated that the instantaneous surface-pressure coefficient would vary from nearly zero to something less than -2, compared with the commonly accepted mean value of about -1.

5. Conclusions

The magnitude of the steady and fluctuating forces acting on a long cylinder of square cross-section are markedly influenced by the presence of large-scale turbulence in the stream. The changes with turbulence include an increase in wake pressure and a reduction in fluctuating lift of about 50 %.

In both smooth and turbulent flow the fluctuating pressures are sufficiently large to warrant attention in regard to both the dynamic response of a structure and the magnitude of instantaneous local pressures on a face.

At an angle of attack of zero the root-mean-square coefficient of fluctuating lift was measured at 1·32 in a smooth stream and 0·68 with large-scale turbulence; the corresponding spanwise correlation lengths were $5{\cdot}6D$ and $3{\cdot}3D$, respectively.

The work described in this report was carried out in the Aerodynamics Division of the National Physical Laboratory, Teddington, and forms the basis of N.P.L. Aero. Report No. 1146. The work was supported in part by the Civil Engineering Research Association. The author wishes to acknowledge the financial assistance afforded by the Eleanor Sophia Wood Travelling Fellowship, 1964, while a Guest Worker at the National Physical Laboratory and on leave from the Department of Civil Engineering, University of Sydney.

Appendix 1. The evaluation of blockage corrections

Mean pressures

Steady pressures and forces on bluff structures may be corrected on the basis of a simple free-streamline model of the flow. From momentum considerations Maskell (1963) has shown that

$$\frac{1-C_p}{1-C_{p_c}} = \frac{C_D}{C_{D_c}} = \frac{k^2}{k_c^2} = 1+\frac{C_D}{k_c^2-1}\frac{A}{C}+O\left(\frac{A}{C}\right)^2,$$

where
$$k^2 = 1-C_{p_b}$$

and the subscript c denotes corrected values;

$$A/C = \text{Model area/Area of tunnel.}$$

Strouhal number

Using the same model employed by Maskell, Roshko (1954) introduced the 'universal Strouhal number' (S_*) based on the 'wake velocity' (U_ω) and wake width (d_ω) such that

$$S_* = nd_\omega/U_\omega = nd_\omega/\{U(1-C_{p_b})^{\frac{1}{2}}\} = nd_\omega/Uk,$$

Roshko has shown experimentally that S_* tends to be a constant and independent of the body causing the wake. If it is then assumed that S_* is independent of constraint we may write,

$$\frac{S}{S_c} = \frac{S_*\dfrac{d}{d_\omega}(1-C_{p_b})^{\frac{1}{2}}}{S_*\dfrac{d}{d_{\omega c}}(1-C_{p_b})^{\frac{1}{2}}},$$

or
$$\frac{S}{S_c} = \frac{d_{\omega c}}{d_\omega}\frac{k_c}{k}.$$

The effect of constraint on wake width is small compared with the effect on k, and Maskell (1963) derives the relationship

$$\frac{d_\omega}{d_{\omega c}} = 1+\frac{C_D-C_{D_c}}{(k^2-1)(k_c^2-1)}\frac{A}{C}.$$

The blockage corrections for Strouhal number can then be written as

$$\frac{S}{S_c} = \frac{k_c}{k}\left\{1+\frac{C_D-C_{D_c}}{(k^2-1)(k_c^2-1)}\frac{A}{C}\right\}.$$

In most cases the bracketed term in this equation is very nearly unity and, very approximately,
$$S/S_c = k_c/k.$$

Fluctuating lift forces (due to vortex shedding)

From considerations of the hydrodynamics of vortex wakes it is possible to derive expressions for C_D, S and C_{L_f} on the basis of the wake model in figure 10.

Ross (1964) summarized the results of a number of workers and gave the following relationships,

$$S = 0{\cdot}28\left(\frac{D}{h}\right)\left(1-\frac{u}{U}\right),$$

$$C_D = 4\sqrt{2}\left(\frac{h}{D}\right)\left(\frac{u}{U}\right)\left(1-0{\cdot}4\frac{u}{U}\right),$$

$$C_{L_f} \simeq \left(\frac{u}{U}\right)\left(\frac{a}{h}\right)\left(\frac{h}{D}\right).$$

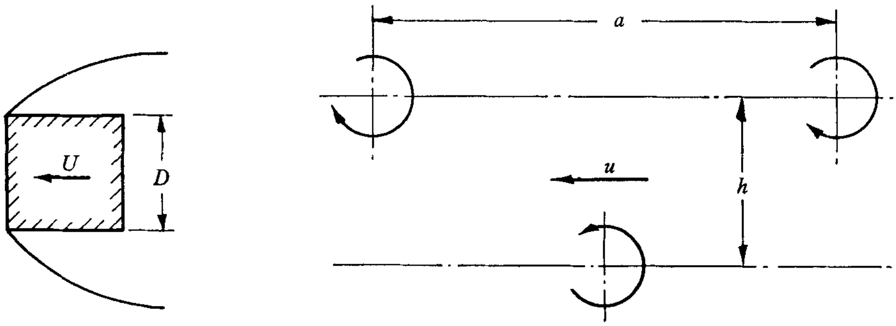

FIGURE 10. Definition sketch—Appendix 1.

On eliminating h/D, we have

$$SC_D = 1{\cdot}58\left(\frac{u}{U}\right)\left(1-\frac{u}{U}\right)\left(1-0{\cdot}4\frac{u}{U}\right),$$

and
$$C_{L_f}/C_D = 0{\cdot}63/\{1-0{\cdot}4(u/U)\}.$$

It is now possible to derive the blockage correction for C_{L_f} from the corrections for S and C_D, but it can be shown that there is little error in assuming that C_{L_f}/C_D is independent of constraint. In the present tests the value of SC_D was approximately 0·25 and hence

$$u/U \simeq 0{\cdot}22,$$

and
$$d(u/U)/d(SC_D) \simeq 1{\cdot}4,$$

or,
$$\frac{(u/U)-(u/U)_c}{(u/U)} = 1{\cdot}4\left\{\frac{SC_D-(SC_D)_c}{SC_D}\right\} = \frac{1{\cdot}4\{k-k_c\}}{k}.$$

For the range of tests covered by this report $(k-k_c)/k$ was no greater than 12 % and hence $(u/U)-(u/U)_c/(u/U)$ was less than 17 %. A correction of this magnitude results in a change in C_{L_f}/C_D of less than 2 % so that the assumption that C_{L_f}/C_D is independent of constraint introduces only minor errors.

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